Quiz 3

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Quiz 3 – *problem1*

$$f_X(x) = \begin{cases} \dfrac{3}{10}, & if\ \ 0 \le x \le 1 \\[2mm] \dfrac{3}{10}, & if\ \ 1 \le x \le 2 \\[2mm] \dfrac{2}{10}, & if\ \ 2 \le x \le 3 \\[2mm] \dfrac{2}{10}, & if\ \ 3 \le x \le 4 \end{cases} \qquad\qquad f_Y(y) = \begin{cases} \dfrac{2}{10}, & if\ \ 0 \le y \le 1 \\[2mm] \dfrac{4}{10}, & if\ \ 1 \le y \le 2 \\[2mm] \dfrac{2}{10}, & if\ \ 2 \le y \le 3 \\[2mm] \dfrac{2}{10}, & if\ \ 3 \le y \le 4 \end{cases}$$

$$E[X] = \int_0^1 \frac{3}{10}x\,dx + \int_1^2 \frac{3}{10}x\,dx + \int_2^3 \frac{2}{10}x\,dx + \int_3^4 \frac{2}{10}x\,dx = \frac{9}{5}$$

$$E[X^2] = \int_0^1 \frac{3}{10}x^2\,dx + \int_1^2 \frac{3}{10}x^2\,dx + \int_2^3 \frac{2}{10}x^2\,dx + \int_3^4 \frac{2}{10}x^2\,dx = \frac{68}{15}$$

$$var(X) = E[X^2] - (E[X])^2 = \frac{97}{75}$$

Quiz 3 – problem2

$$X = max\{x_1, x_2\}$$

$$P_X(k) = F_X(k) - F_X(k-1)$$

$$\text{where} \quad F_X(k) = P(x_1 \leq k)P(x_2 \leq k) = \frac{k}{5} \cdot \frac{k-1}{4} = \frac{k^2 - k}{20}$$

$$\therefore P_X(k) = \frac{k^2 - k}{20} - \frac{(k-1)^2 - (k-1)}{20} = \frac{x-1}{10}$$

$$\Rightarrow P_X(k) = \begin{cases} \dfrac{x-1}{10} & , x = 2,3,4,5 \\ 0 & , otherwise \end{cases}$$

Quiz 3 – problem3

$$\int_0^k \lambda e^{-\lambda x}\,dx = \frac{1}{2}$$

$$\int_0^k \lambda e^{-\lambda x}\,dx = -e^{-\lambda x}\Big|_0^k = -e^{-\lambda k} + 1 = \frac{1}{2}$$

$$\therefore e^{-\lambda k} = \frac{1}{2} \quad , \quad -\lambda k = ln\left(\frac{1}{2}\right) = -ln\,2 \quad , \quad \lambda k = ln\,2$$

$$k = \frac{ln2}{\lambda} \quad , \quad \lambda = \frac{ln2}{k}$$

Quiz 3 – *problem4&5*

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$$mean = 15 \ , \ \sigma^2 = 100 \ , \ \sigma = 10$$

$$P(-5 \leq X \leq 35) = \phi\left(\frac{35-15}{10}\right) - \phi\left(\frac{-5-15}{10}\right)$$

$$= \phi(2) - \phi(-2) = 0.9772 - (1 - 0.9772) = 0.9544$$

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$$(i)\frac{7}{10}$$

$$(ii)\frac{3}{8}$$