Numerical Methods

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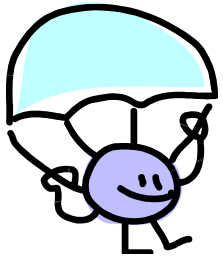
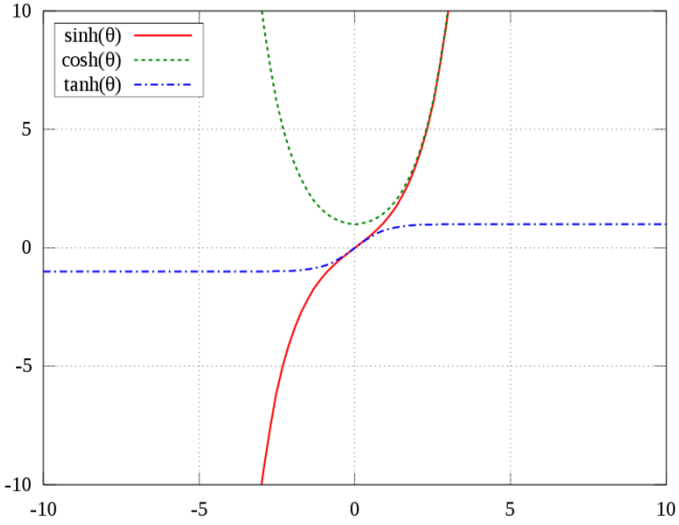
What are Numerical Methods? (1/3)

- Numerical methods are techniques by which mathematical problems are formulated so that they can be solved by arithmetic and logic operations
  - Because computers excel at performing such operations, numerical methods are sometimes referred to as **computer mathematics**
  - The role of numerical methods in engineering problem solving has increased dramatically in recent years



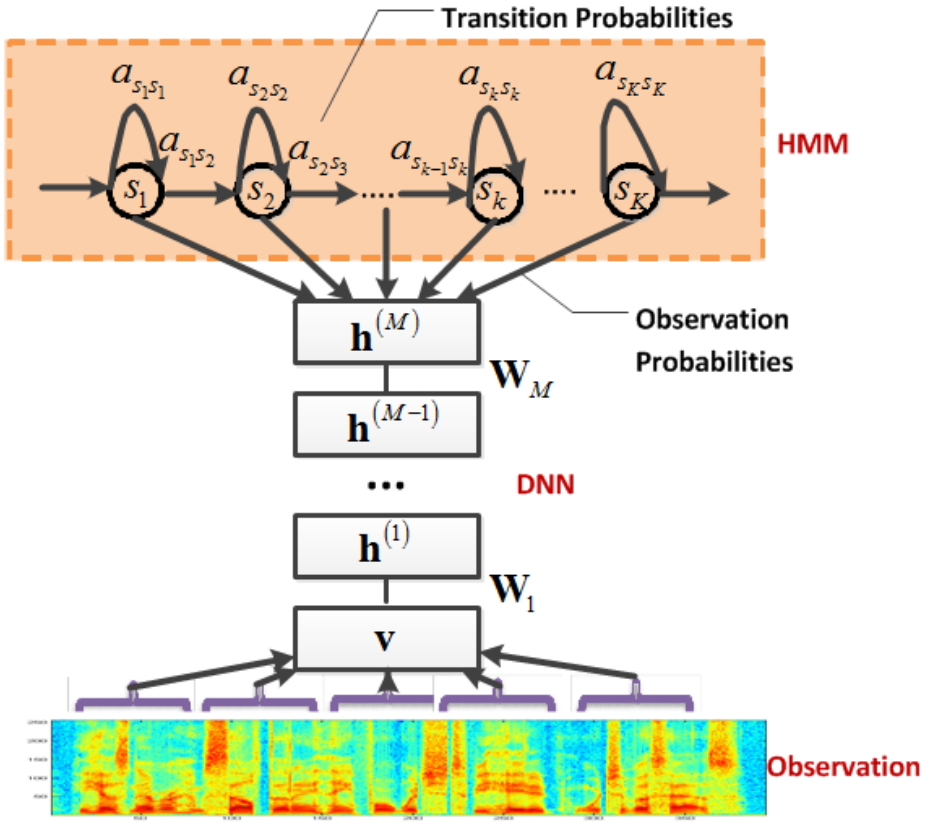
$$v(t_{i+1}) = \sqrt{\frac{gm}{c_d}} \tanh\left(\sqrt{\frac{gc_d}{m}} t_{i+1}\right)$$

$$\Rightarrow v(t_{i+1}) = v(t_i) + \left[g - \frac{c_d}{m}[v(t_i)]^2\right](t_{i+1} - t_i)$$

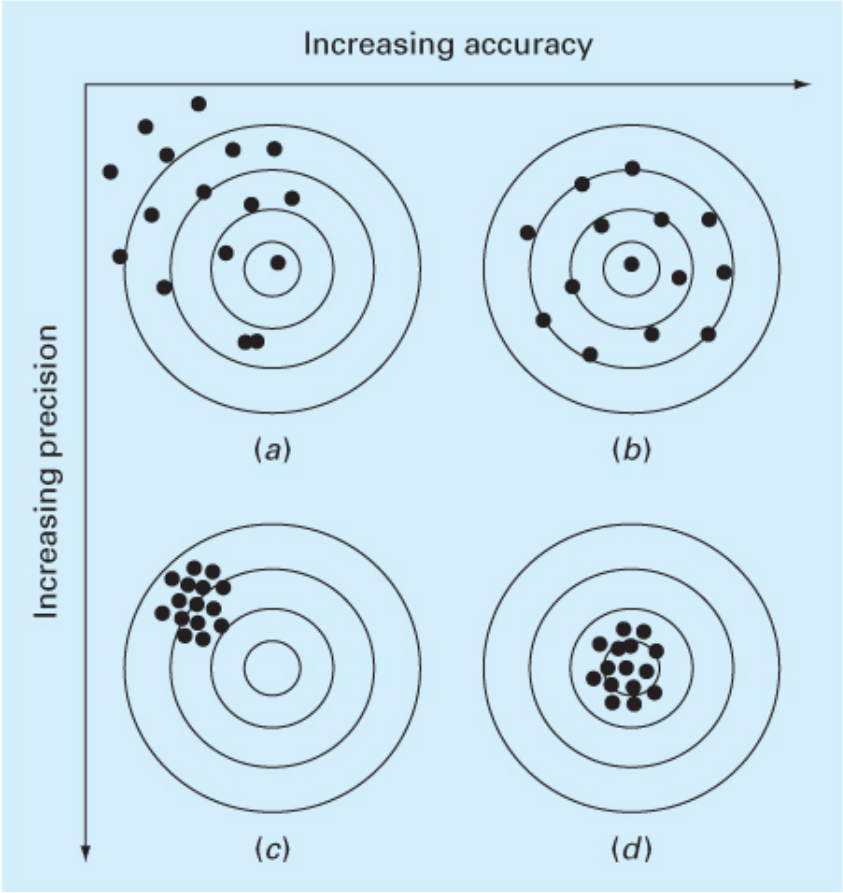


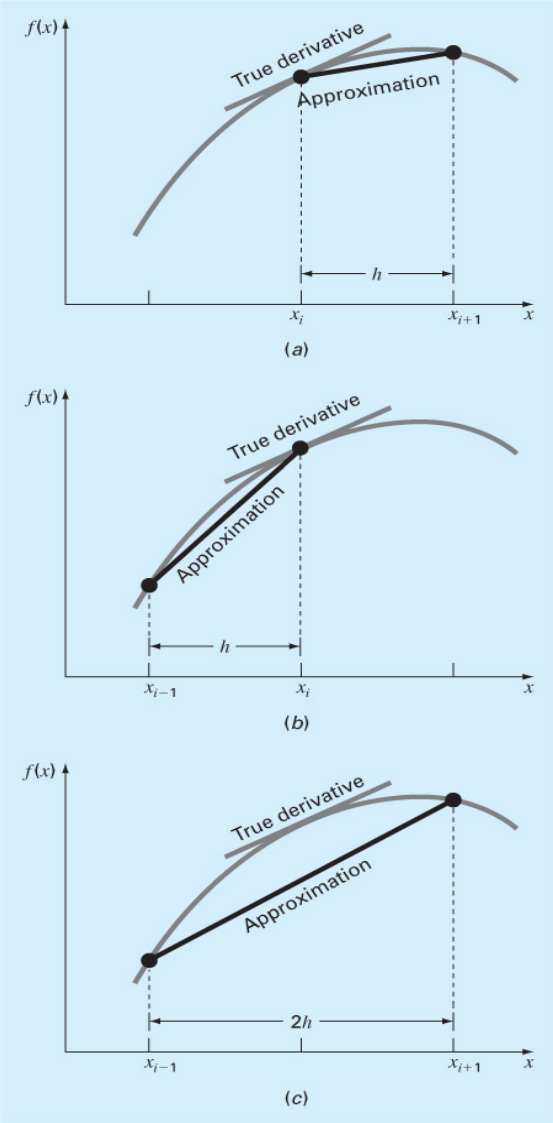
What are Numerical Methods? (2/3)

- E.g., complicated mathematical problems in speech recognition



What are Numerical Methods? (3/3)





Why Studying Numerical Methods? (1/2)

- Numerical methods greatly expand the types of problems you can address
  - E.g., handling large systems of equations, nonlinearities, and complicated geometries
  - As such, greatly enhance your problem-solving sills
- Numerical methods allow you to use “canned” software packages with insight rather than treating them as “black boxes”
  - Enable intelligent use of these packages by an understanding the basic theory underlying the methods
- Many problems cannot be approached using canned programs
  - With the help of numerical methods, you can design your programs to solve problems

Why Studying Numerical Methods? (2/2)

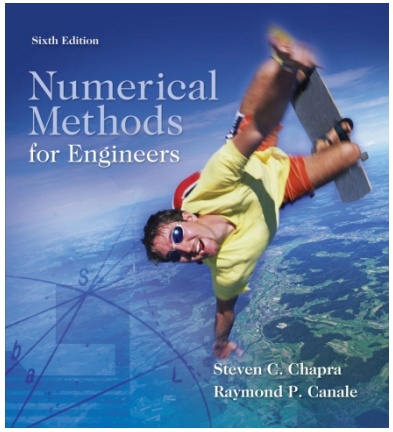
- Numerical methods are an efficient vehicle for learning to use computers and can also illustrate the power and limitations of computers
  - We should learn to acknowledge and control the errors of approximation that are part and parcel of large-scale numerical methods
- Numerical methods provide a vehicle for you to reinforce your understanding of mathematics
  - One function of numerical methods is to reduce higher mathematics to basic arithmetic operations. In so doing, we can enhance our understanding and insight of complicated problems

Objectives of this Course

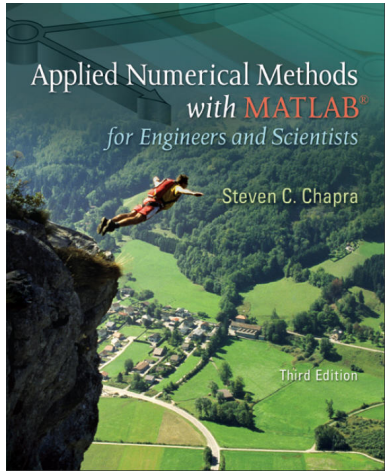
- This course sets out to introduce the fundamentals underpinning numerical methods
- Learn to decide how and when to choose a numerical method, and how to implement and apply it to solve practical problems in science and engineering
- Understand how numerical methods and computers work in tandem to generate reliable solutions to mathematical problems
- Have a solid appreciation of the capabilities and limitations of various numerical methods

Textbooks

- Steven C. Chapra, ***Applied Numerical Methods with MATLAB for Engineers and Scientists***, McGraw-Hill, 2012



- Steven C. Chapra and Raymond P. Canale, ***Numerical Methods for Engineers***, McGraw-Hill, 2010

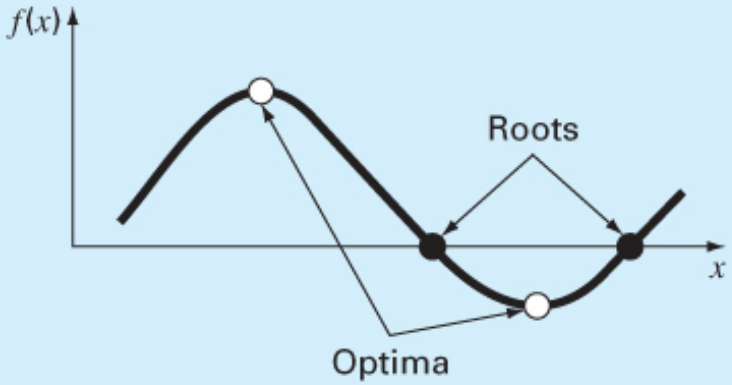


Some Numerical Methods (1/2)

(a) *Part 2*: Roots and optimization

Roots: Solve for x so that $f(x) = 0$

Optimization: Solve for x so that $f'(x) = 0$

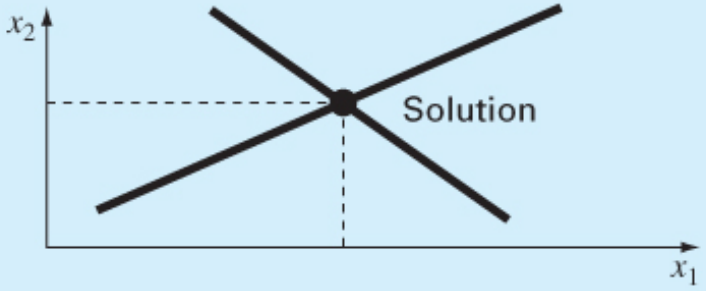


(b) *Part 3*: Linear algebraic equations

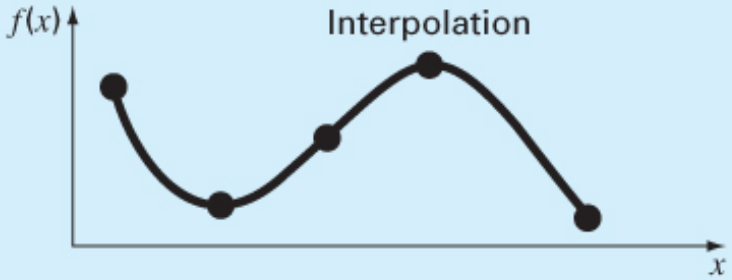
Given the a's and the b's, solve for the x's

$a_{11}x_1 + a_{12}x_2 = b_1$

$a_{21}x_1 + a_{22}x_2 = b_2$



(c) *Part 4*: Curve fitting

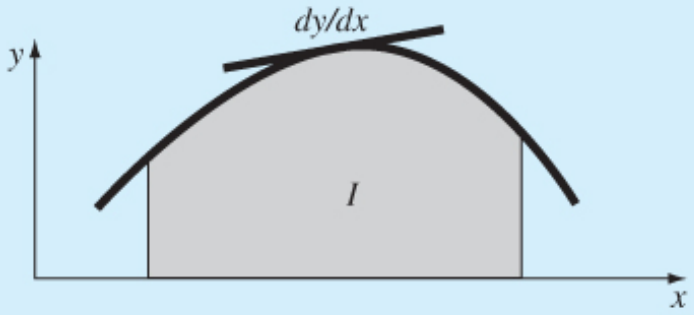


Some Numerical Methods (2/2)

(d) *Part 5*: Integration and differentiation

Integration: Find the area under the curve

Differentiation: Find the slope of the curve



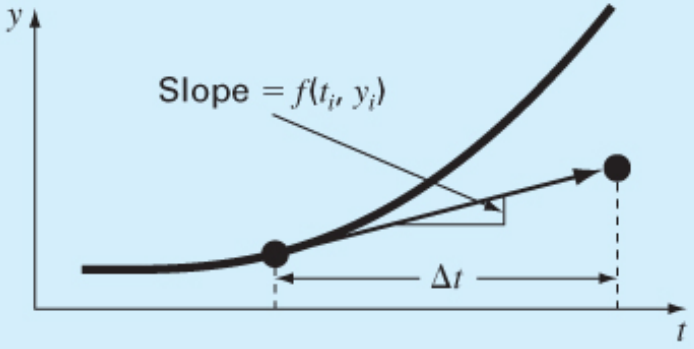
(e) *Part 6*: Differential equations

Given

$$\frac{dy}{dt} \approx \frac{\Delta y}{\Delta t} = f(t, y)$$

solve for y as a function of t

$$y_{i+1} = y_i + f(t_i, y_i)\Delta t$$



Grading (Tentative)

- Midterm and Final: 45%
- Homework/Projects: 40%
- Attendance/Other: 15%