Linear Algebra
Quiz 5

11:10 a.m. - 12:10 a.m., January 8, 2016

Note: You have to answer the questions with supporting explanations (i.e., show all your work) if needed.

1. 1. If $T(x_1, x_2, x_3) = (x_1 + 3x_3, 3x_1 - 2x_2)$, then
   (i) Find the domain and codomain of *T*. (15%)
   (ii) Find the image of $\mathbf{x} = (1, -1, 2)$ under *T*. (15%)
   Ans. (i) domain: R^3; codomain: R^2 (ii) $\begin{bmatrix} 7 \\ 5 \end{bmatrix}$

2. (i) Find the standard matrix *A* for the reflection about the line *y*=*x*. (15%)
   (ii) Find the standard matrix *B* for the orthogonal projection on the line *y*=*x*. (15%)
   Ans. (i) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ (ii) $\begin{bmatrix} 1/2 & 1/2 \\ 1/2 & 1/2 \end{bmatrix}$

3. Consider a transformation matrix *C* shown below:

$$C = \begin{bmatrix} 2 & 4 \\ 3 & 5 \end{bmatrix}$$

   (i) Express *C* as a product of elementary matrices, and then describe the effect on R^2 of multiplication *C* in terms of expansions (or compressions), reflections and shearing. (15%)
   (ii) Find an equation of the image of the line $y = 2x + 3$ under multiplication by *C*. (15%)
   (iii) Find the area (面積) of the image of the triangle with vertices (0, 0), (6, 0), (3, 3) under multiplication by *C*. (10%)
   Ans. (i) For example, $C = E_1^{-1}E_2^{-1}E_3^{-1}E_4^{-1} = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$, where the effect of multiplying by C is equivalent to
   1. (E_4^{-1}) shearing by a factor of 2 in the *x*-direction.
   2. (E_3^{-1}) then reflecting about the *x*-axis.
   3. (E_2^{-1}) then shearing by a factor of 3 in the *y*-direction.
   4. (E_1^{-1}) then expending by a factor of 2 in the *x*-direction.

   (ii) $10y = 13x - 6$
   (iii) the area of the image is equal to the absolute value of the determinant of *C* multiplied by the area of the original triangle => $|\det(C)| \times \text{area of the original triangle} = 2 \times 9 = 18$