Linear Algebra
Homework 3

Due date: 2016/01/08

Note: You have to answer the questions with supporting explanations if needed. The computations have to be accomplished with paper and pencil.

1. If $T(x_1, x_2, x_3) = (x_1 + 3x_2, x_1 - 3x_2)$, then
   (i) Find the domain and codomain of T; (ii) Find the image of $\mathbf{x} = (2, -2, 5)$ under T.
2. Use matrix multiplication to find the reflection of (3, 2) about
   (i) the x-axis; (ii) the y-axis; (iii) the line y=x.
3. Describe the geometric effect of multiplying a vector $\mathbf{x}$ by a matrix A.

$$\text{(i) } A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}; \text{ (ii) } A = \begin{bmatrix} \sqrt{3}/2 & -1/2 \\ 1/2 & \sqrt{3}/2 \end{bmatrix}$$

4. Let $T_1(x_1, x_2, x_3) = (x_1 - x_2, x_1 + 3x_2, 0)$ and $T_2(x_1, x_2, x_3) = (2x_2, x_1 - 5x_2, x_1)$.
   (i) Find the standard matrices for $T_2 \circ T_1$ and $T_1 \circ T_2$.
   (ii) Use the matrices obtained in part (i) to find the formulas for $T_2(T_1(x_1, x_2, x_3))$ and $T_1(T_2(x_1, x_2, x_3))$.
5. Find the standard matrix for the following matrix operators.
   (i) $T: R^3 \to R^3$ reflects a vector about the xz-plane and then contracts that vector by a factor of 1/2.
   (ii) $T: R^3 \to R^3$ projects a vector orthogonally onto the xz-plane and then projects that vector orthogonally onto the xy-plane

6. Given a matrix $A = \begin{bmatrix} 2 & 0 \\ 2 & -1 \end{bmatrix}$
   (i) express A as a product of elementary matrices, and then describe the effect on R^2 of multiplication by A in terms of compression, expansions, reflections and shears.
   (ii) Sketch the image of the rectangle with vertices (0, 0), (1, 0), (1, 2) and (0, 2) under the transformation by A.

$$A=\begin{bmatrix}2 & 0\\ 2 & -1\end{bmatrix}=\begin{bmatrix}2 & 0\\ 0 & 1\end{bmatrix}\begin{bmatrix}1 & 0\\ 2 & 1\end{bmatrix}\begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix}$$