Linear Algebra

Berlin Chen

Department of Computer Science & Information Engineering

National Taiwan Normal University

Linear Algebra

- Linear algebra is a branch of mathematics and continues to figure prominently in computer science and electrical engineering
  - Computation, geometry, theory, practical applications, to name just a few

- Simply put, linear algebra is the study of vectors, matrices, vector spaces and linear transformations

$$a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = b_1$$
$$a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n = b_2$$
$$\vdots \qquad \vdots \qquad \qquad \vdots \qquad \vdots$$
$$a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n = b_m$$

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} & b_1 \\ a_{21} & a_{22} & \cdots & a_{2n} & b_2 \\ \vdots & \vdots & & \vdots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} & b_m \end{bmatrix}$$

Main Objectives

- Develop the definitions, concepts and theories associated with linear algebra
  - Fundamentals: vectors operations, matrices operations, determinants, Euclidean vector spaces, linear systems, etc.
  - Advanced topics: matrix diagonalization, matrix factorization, linear transforms, numerical methods, practical applications, etc.
- Learn to make effective use of linear algebra in dealing with practical issues of interest
  - E.g., multimedia (text, speech, music and image) processing

Information Retrieval (IR): Measuring Degree of Similarity between Query and Documents

$q = 0u_1 + 0u_2 + 3u_3$
$d_1 = 2u_1 + 4u_2 + 5u_3$
$d_2 = 3u_1 + 7u_2 + 7u_3$

$d_1 = 2u_1 + 4u_2 + 5u_3$

$d_2 = 3u_1 + 7u_2 + 7u_3$

$q = 0u_1 + 0u_2 + 3u_3$

$sim(d_j, q)$

$= cosine\ (\Theta)$

$= \dfrac{\vec{d}_j \bullet \vec{q}}{|\vec{d}_j| \times |\vec{q}|}$

$= \dfrac{\sum_{i=1}^{t} w_{i,j} \times w_{i,q}}{\sqrt{\sum_{i=1}^{t} w_{i,j}^2} \times \sqrt{\sum_{i=1}^{t} w_{i,q}^2}}$
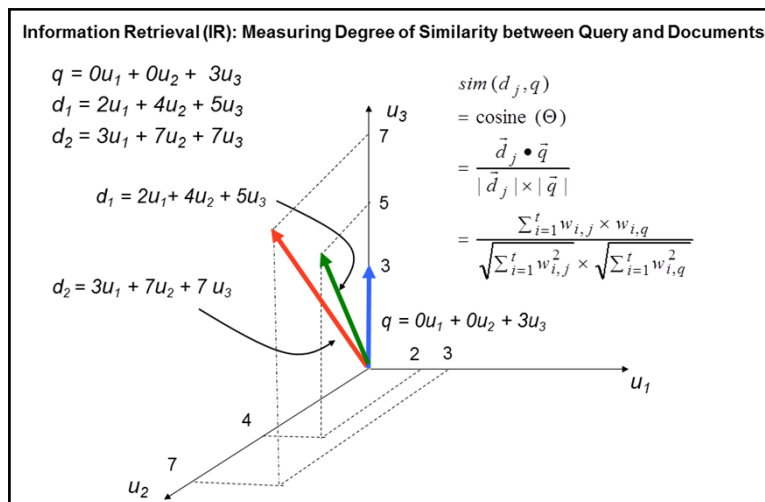
- Start with a matrix describing the intra- and Inter-document statistics between all terms and all documents
- Singular value decomposition (SVD) is then performed on the matrix to project all term and document vectors onto a reduced latent topical space

Documents
Words
A U Σ V^T latent semantic space A'
mxn k mxr kxk rxr rxn mxn
latent semantic space
$r \leq min(m,n)$

$\|A\|_F^2 = \sum_{i=1}^{m} \sum_{j=1}^{n} a_{ij}^2 \implies \|A\|_F^2 = \sigma_1^2 + \sigma_2^2 + ... + \sigma_r^2$?

- In the context of information retrieval (IR), matching between queries and documents can be carried out in this topical space

Textbook & Course Website

- H. Anton, C. Rorres, *Elementary Linear Algebra with Supplemental Applications*, 11th edition, Wiley, 2014
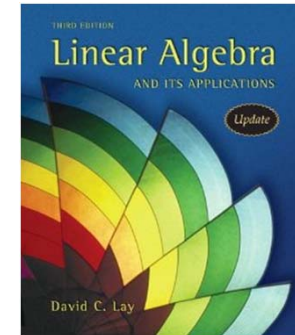
  – Website

    http://www.wiley.com/WileyCDA/WileyTitle/productCd-1118677455.html
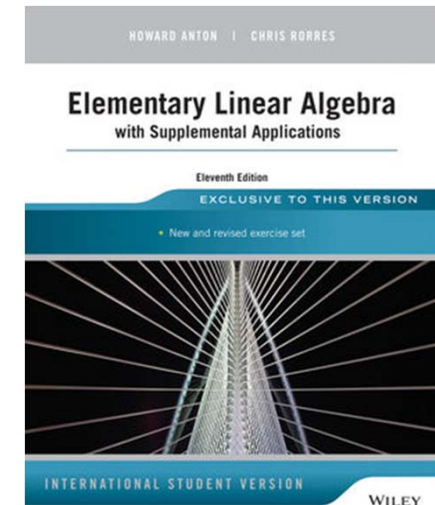
- Course Website

    http://berlin.csie.ntnu.edu.tw/Courses/LinearAlgebra/2014F-LA_Main.htm

Reference Books

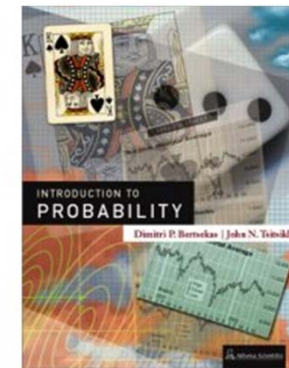- D. C. Lay, *Linear Algebra and Its Applications, 3rd Updated Edition*, Addison Wesley, 2005
  - Website

    http://www.laylinalgebra.com/

- D. P. Bertsekas, J. N. Tsitsiklis, *Introduction to Probability*, 2nd Edition, Athena Scientific, 2008
  - Website

    http://www.athenasc.com/probbook.html

Tentative Topic List

1.		Systems of Linear Equations and Matrices
2.		Determinants
3.		Euclidean Vector Spaces
4.		General Vector Spaces
5.		Eigenvalues and Eigenvectors
6.		Inner Product Spaces
7.		Diagonalization and Quadratic Forms
8.		Linear Transformations

Grading (*Tentatively!*)

- Midterm and Final: 45%

- Quizzes (≧ 5 times) and Homework: 45%

- Attendance/Other: 10%

- TA：洪孝宗同學 (語音實驗室；R208)
  - E-mail: 60047064S@ntnu.edu.tw
  - Tel: 7734-6676