Models for Information Retrieval

- Fuzzy Set, Extended Boolean, Generalized Vector Space Models

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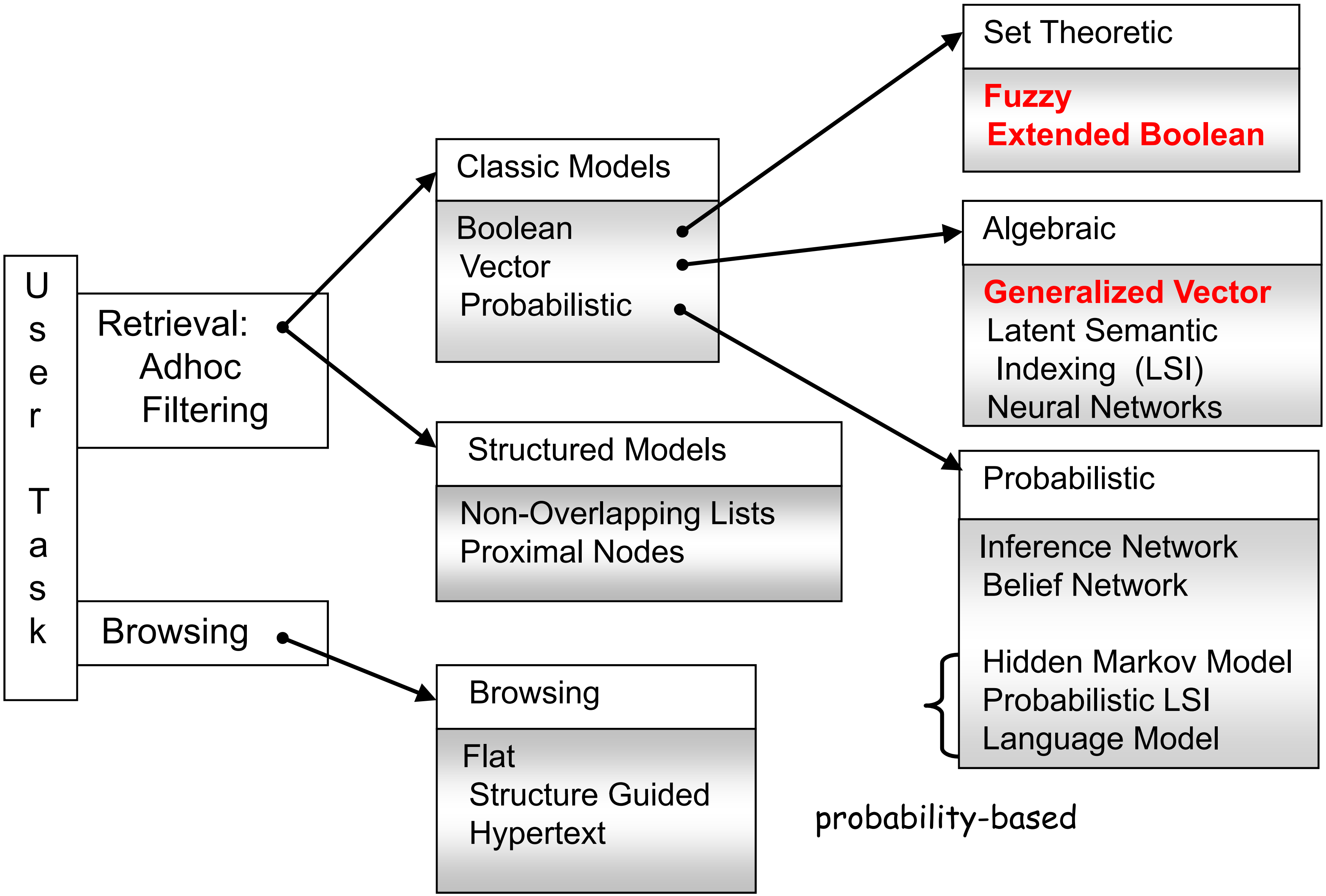
Reference:

1. *Modern Information Retrieval*. Chapter 2

Taxonomy of Classic IR Models

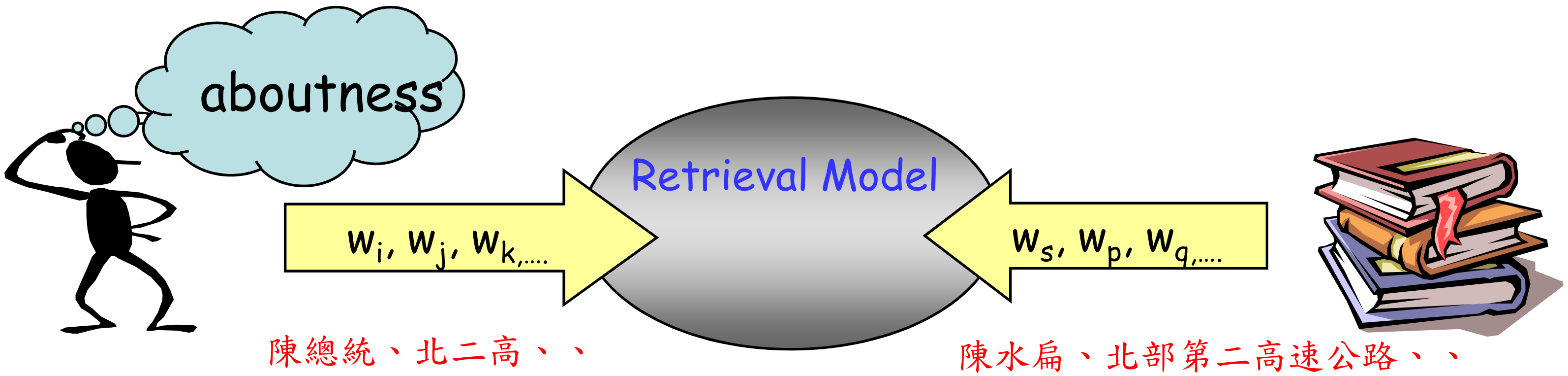


Outline

- Alternative Set Theoretic Models
  - Fuzzy Set Model (Fuzzy Information Retrieval)
  - Extended Boolean Model
- Alternative Algebraic Model
  - Generalized Vector Space Model

Fuzzy Set Model

- Premises
  - Docs and queries are represented through sets of keywords, therefore the matching between them is vague
    - Keywords cannot completely describe the user's information need and the doc's main theme



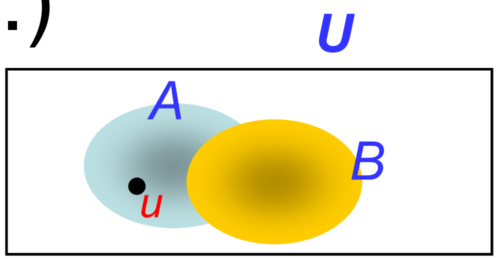
  - For each query term (keyword)
    - Define a fuzzy set and that each doc has a degree of membership (0~1) in the set

Fuzzy Set Model (cont.)

- Fuzzy Set Theory
  - Framework for representing classes (sets) whose boundaries are not well defined
  - Key idea is to introduce the notion of a *degree of membership* associated with the elements of a set
  - This degree of membership varies from 0 to 1 and allows modeling the notion of *marginal* membership
    - 0 →no membership
    - 1 →full membership
  - Thus, membership is now a gradual instead of abrupt
    - Not as conventional Boolean logic

Here we will define a fuzzy set for each query (or index) term, thus each doc has a degree of membership in this set.

Fuzzy Set Model (cont.)



- Definition
  - A fuzzy subset A of a universal of discourse U is characterized by a membership function μ_A: $U \rightarrow [0,1]$
    - Which associates with each element u of U a number $\mu_A(u)$ in the interval [0,1]
  - Let A and B be two fuzzy subsets of U. Also, let $\overline{A}$ be the complement of A. Then,
    - Complement $\mu_{\overline{A}}(u) = 1 - \mu_A(u)$
    - Union $\mu_{A \cup B}(u) = \max(\mu_A(u), \mu_B(u))$
    - Intersection $\mu_{A \cap B}(u) = \min(\mu_A(u), \mu_B(u))$

Fuzzy Set Model (cont.)

- Fuzzy information retrieval

Defining term relationship

  - Fuzzy sets are modeled based on a **thesaurus**
  - This thesaurus can be constructed by a term-term correlation matrix (or called keyword connection matrix)
    - $\vec{c}$: a term-term correlation matrix
    - $c_{i,l}$: a normalized correlation factor for terms k_i and k_l

$$c_{i,l} = \frac{n_{i,l}}{n_i + n_l - n_{i,l}}$$

ranged from 0 to 1

n_i : no of docs that contain k_i
$n_{i,l}$: no of docs that contain both k_i and k_l

docs, paragraphs, sentences, ..

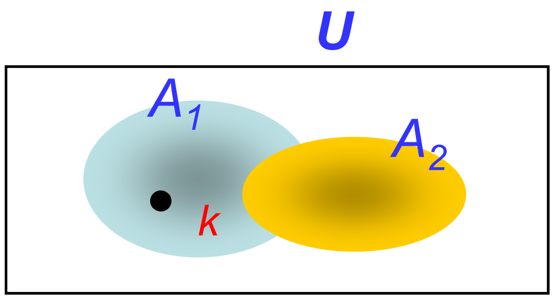
    - We now have the notion of proximity among index terms

  - The relationship is symmetric !

$$\mu_{k_i}(k_l) = c_{i,l} = c_{l,i} = \mu_{k_l}(k_i)$$

Fuzzy Set Model (cont.)

- The union and intersection operations are modified here



$$
\begin{aligned}
& ab + \bar{a}b + a\bar{b} \\
&= ab + (1-a)b + a(1-b) \\
&= ab + b - ab + a - ab \\
&= 1 - (1 - a - b + ab) \\
&= 1 - (1-a)(1-b)
\end{aligned}
$$

  - Union: algebraic sum (instead of max)

$$
\begin{aligned}
\mu_{A_1 \cup A_2}(k) &= \mu_{A_1}(k)\mu_{A_2}(k) + \mu_{\overline{A_1}}(k)\mu_{A_2}(k) + \mu_{A_1}(k)\mu_{\overline{A_2}}(k) \\
&= 1 - \prod_{j=1}^{2}\left(1 - \mu_{A_j}(k)\right)
\end{aligned}
$$

a negative algebraic product

$$
\Rightarrow \quad \mu_{A_1 \cup A_2 \cdots \cup A_n}(k) = \mu_{\cup_j A_j}(k) = 1 - \prod_{j=1}^{n}\left(1 - \mu_{A_j}(k)\right)
$$

  - Intersection: algebraic product (instead of min)

$$
\mu_{A_1 \cap A_2}(k) = \mu_{A_1}(k)\mu_{A_2}(k) \quad \Rightarrow \quad \mu_{A_1 \cap A_2 \cdots \cap A_n}(k) = \prod_{j=1}^{n} \mu_{A_j}(k)
$$

Fuzzy Set Model (cont.)

– The degree of membership between a doc d_j and an index term k_i

algebraic sum (a doc is a union of index terms)

	k_a	k_b
k_i	$c_{i,a}$	$c_{i,b}$
	$1-c_{i,a}$	$1-c_{i,b}$

$$\mu_{k_i}(d_j) = \boxed{\mu_{d_j}(k_i) = \mu_{\underset{k_l \in d_j}{\cup} k_l}(k_i)}$$

$$= 1 - \prod_{k_l \in d_j}\left(1 - \mu_{k_l}(k_i)\right) = 1 - \prod_{k_l \in d_j}\left(1 - c_{i,l}\right)$$

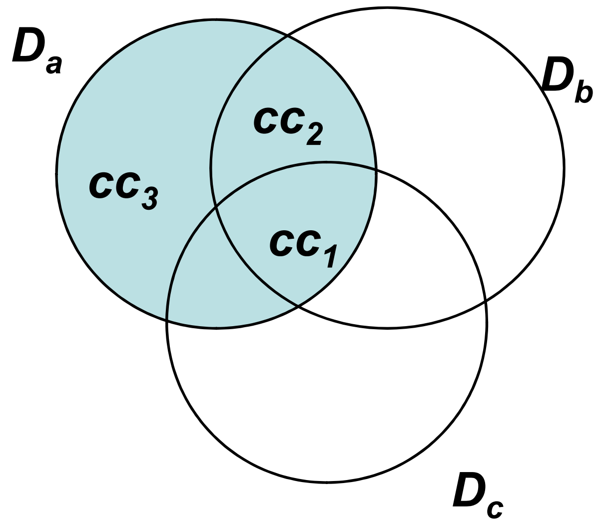
- Computes an **algebraic sum** over all terms in the doc d_j
  - Implemented as the complement of a negative algebraic product
  - A doc d_j belongs to the fuzzy set associated to the term k_i if its own terms are related to k_i
- If there is at least one index term k_l of d_j which is strongly related to the index k_i ($c_{i,l} \sim 1$) then $\mu_{k_i,d_j} \sim 1$
  - k_i is a good fuzzy index for doc d_j
  - *And vice versa*

Fuzzy Set Model (cont.)

- Example:
  - Query $q=k_a \wedge (k_b \vee \neg k_c)$ disjunctive normal form

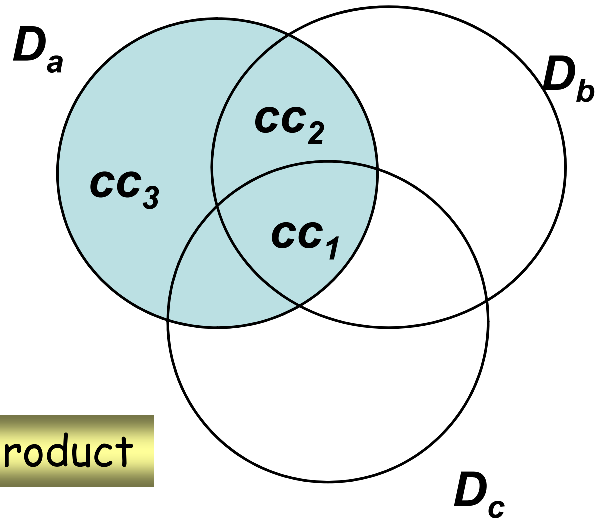
    $\vec{q}_{dnf}=(k_a \wedge k_b \wedge k_c) \vee (k_a \wedge k_b \wedge \neg k_c) \vee (k_a \wedge \neg k_b \wedge \neg k_c)$

    $=cc_1+cc_2+cc_3$ conjunctive component



  - D_a is the fuzzy set of docs associated to the term k_a
  - Degree of membership ?

Fuzzy Set Model (cont.)



- Degree of membership

algebraic sum

$$\mu_q(d_j)=\mu_{cc_1\cup cc_2\cup cc_3}(d_j)$$

for a doc d_j in the fuzzy answer set D_q

negative algebraic product

$$=1-\prod_{i=1}^{3}\left(1-\mu_{cc_i}(d_j)\right)$$

$$=1-\left(1-\overbrace{\mu_{a\cap b\cap c}}^{cc_1}(d_j)\right)\left(1-\overbrace{\mu_{a\cap b\cap \bar{c}}}^{cc_2}(d_j)\right)\left(1-\overbrace{\mu_{a\cap \bar{b}\cap \bar{c}}}^{cc_3}(d_j)\right)$$

algebraic product

$$=1-\left(1-\mu_a(d_j)\mu_b(d_j)\mu_c(d_j)\right)$$
$$\times\left(1-\mu_a(d_j)\mu_b(d_j)(1-\mu_c(d_j))\right)\times\left(1-\mu_a(d_j)(1-\mu_b(d_j))(1-\mu_c(d_j))\right)$$

More on Fuzzy Set Model

- Advantages
  - The correlations among index terms are considered
  - Degree of relevance between queries and docs can be achieved

- Disadvantages
  - Fuzzy IR models have been discussed mainly in the literature associated with fuzzy theory
  - Experiments with standard test collections are not available
  - Do not consider the frequecny (or counts) of a term in a document or a query

Extended Boolean Model

Salton et al., 1983

- Motive
  - Extend the Boolean model with the functionality of partial matching and term weighting
    陳水扁 及 呂秀蓮
    - E.g.: in Boolean model, for the qery $q=k_x \wedge k_y$, a doc contains either k_x or k_y is as irrelevant as another doc which contains neither of them
    - How about the disjunctive query $q=k_x \vee k_y$ 陳水扁 或 呂秀蓮
  - Combine Boolean query formulations with characteristics of the vector model
    - Term weighting
    - Algebraic distances for similarity measures

a ranking can be obtained

Extended Boolean Model (cont.)

- Term weighting
  - The weight for the term k_x in a doc d_j is

$$w_{x,j} = tf_{x,j} \times \boxed{\frac{idf_x}{\max_i idf_i}}$$

ranged from 0 to 1

normalized frequency

Normalized *idf*

    - $w_{x,j}$ is normalized to lie between 0 and 1

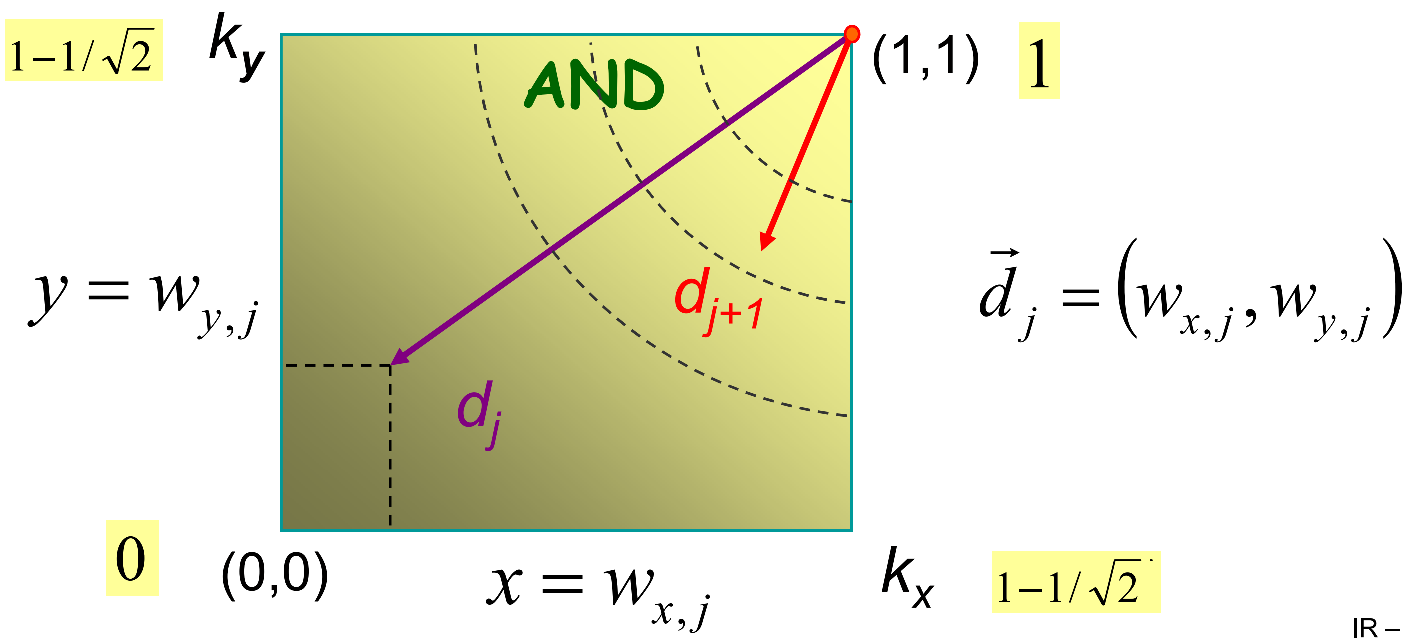
- Assume two index terms k_x and k_y were used
  - Let x denote the weight $w_{x,j}$ of term k_x on doc d_j
  - Let y denote the weight $w_{y,j}$ of term k_y on doc d_j
  - The doc vector $\vec{d}_j = (w_{x,j}, w_{y,j})$ is represented as $d_j = (x, y)$
  - Queries and docs can be plotted in a two-dimensional map

Extended Boolean Model (cont.)

- If the query is $q=k_x \wedge k_y$ (conjunctive query)
  - -The docs near the point (1,1) are preferred
  - -The similarity measure is defined as

$$sim(q_{and}, d) = 1 - \sqrt{\frac{(1-x)^2 + (1-y)^2}{2}}$$

2-norm model
(Euclidean distance)

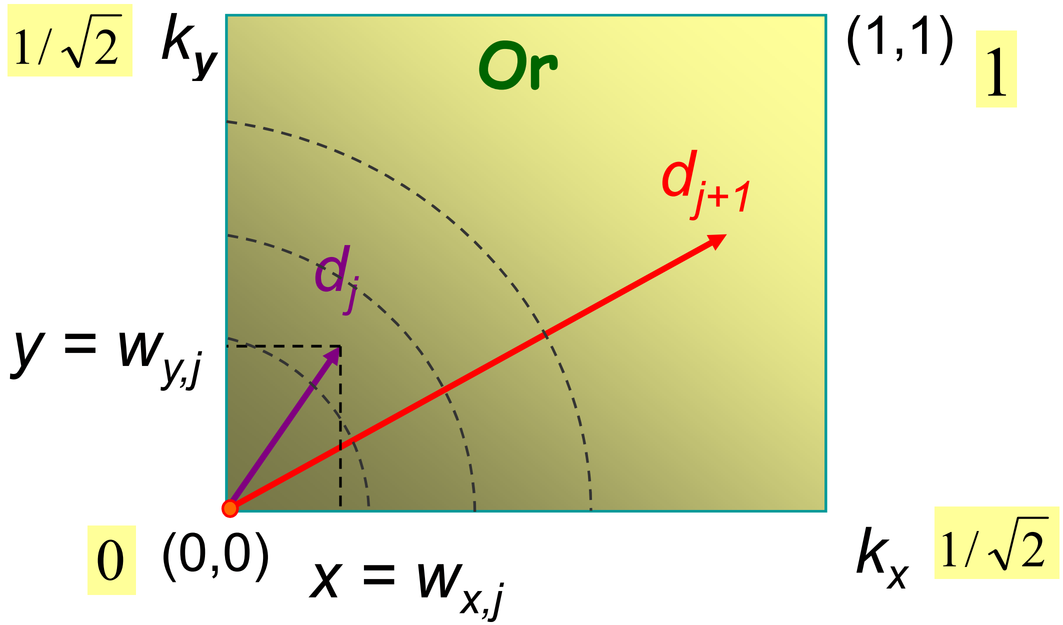


Extended Boolean Model (cont.)

- If the query is $q=k_x \vee k_y$ (disjunctive query)
  - -The docs far from the point (0,0) are preferred
  - -The similarity measure is defined as

$$sim(q_{or}, d) = \sqrt{\frac{x^2 + y^2}{2}}$$

2-norm model
(Euclidean distance)



Extended Boolean Model (cont.)

- The similarity measures $sim(q_{or}, d)$ and $sim(q_{and}, d)$ also lie between 0 and 1

Extended Boolean Model (cont.)

- Generalization
  - *t* index terms are used → *t*-dimensional space
  - *p*-norm model, $1 \leq p \leq \infty$

$$q_{and} = k_1 \wedge^p k_2 \wedge^p \ldots \wedge^p k_m \Rightarrow sim(q_{and}, d) = 1 - \left(\frac{(1-x_1)^p + (1-x_2)^p + \ldots + (1-x_m)^p}{m} \right)^{\frac{1}{p}}$$

$$q_{or} = k_1 \vee^p k_2 \vee^p \ldots v^p k_m \Rightarrow sim(q_{or}, d) = \left(\frac{x_1^{\ p} + x_2^{\ p} + \ldots + x_m^{\ p}}{m} \right)^{\frac{1}{p}}$$

  - Some interesting properties

Similar to vector space model

- *p=1* $\Rightarrow sim(q_{and}, d) = sim(q_{or}, d) = \dfrac{x_1 + x_2 + \ldots + x_m}{m}$
- *p=*∞ $\Rightarrow sim(q_{and}, d) \approx \min(x_i)$

  $sim(q_{or}, d) \approx \max(x_i)$

just like the formula of fuzzy logic

Extended Boolean Model (cont.)

- Example query 1: $q = (k_1 \wedge^p k_2) \vee^p k_3$
  - Processed by grouping the operators in a predefined order

$$sim(q,d) = \left(\frac{\left(1 - \left(\frac{(1-x_1)^p + (1-x_2)^p}{2}\right)^{\frac{1}{p}}\right)^p + x_3^p}{2} \right)^{\frac{1}{p}}$$

- Example query 2: $q = (k_1 \vee^2 k_2) \wedge^\infty k_3$
  - Combination of different algebraic distances

$$sim(q,d) = \min\left(\left(\frac{x_1^2 + x_2^2}{2}\right)^{\frac{1}{2}}, x_3 \right)$$

More on Extended Boolean Model

- Advantages
  - A hybrid model including properties of both the set theoretic models and the algebraic models
  - That is, relax the Boolean algebra by interpreting Boolean operations in terms of algebraic distances
    - By varying the parameter *p* between 1 and infinity, we can vary the *p* -norm ranking behavior from that of a vector-like ranking to that of a fuzzy logic-like ranking
    - Have the possibility of using combinations of different values of the parameter *p* in the same query request

More on Extended Boolean Model (cont.)

- Disadvantages
  - *Distributive operation* does not hold for ranking computation
    - E.g.:

$$q_1 = \left(k_1 \wedge^2 k_2\right) \vee^2 k_3, q_2 = \left(k_1 \vee^2 k_3\right) \wedge^2 \left(k_2 \vee^2 k_3\right)$$

$$\left(\frac{\left(1 - \left(\frac{(1-x_1)^2 + (1-x_2)^2}{2} \right)^{\frac{1}{2}} \right)^2 + x_3^2}{2} \right)^{\frac{1}{2}} \qquad sim\left(q_1, d\right) \neq sim\left(q_2, d\right) \qquad 1 - \left(\frac{\left(1 - \left(\frac{x_1^2 + x_2^2}{2} \right) \right)^2 + \left(1 - \left(\frac{x_2^2 + x_3^2}{2} \right) \right)^2}{2} \right)^{\frac{1}{2}}$$

  - Assumes mutual independence of index terms

Generalized Vector Model

Wong et al., 1985

- Premise
  - Classic models enforce independence of index terms
  - For the **Vector model**
    - Set of term vectors $\{\vec{k}_1, \vec{k}_1, \ldots, \vec{k}_t\}$ are linearly independent and form a basis for the subspace of interest
    - Frequently, it means pairwise orthogonality

      $\forall i,j \Rightarrow \vec{k}_i \cdot \vec{k}_j = \vec{0}$ (in a more restrictive sense)
- Wong et al. proposed an interpretation
  - An alternative intepretation: The index term vectors are linearly independent, but not pairwise orthogonal
    - Generalized Vector Model

Generalized Vector Model (cont.)

- **Key idea**
  - Index term vectors form the basis of the space are not orthogonal and are represented in terms of smaller components (minterms)
- **Notations**
  - $\{k_1, k_2, ..., k_t\}$: the set of all terms
  - $w_{i,j}$: the weight associated with $[k_i, d_j]$
  - **Minterms**:binary indicators (0 or 1) of all patterns of occurrence of terms within documents
    - Each represent one kind of co-occurrence of index terms in a specific document

Generalized Vector Model (cont.)

- Representations of **minterms**

m_1=(0,0,….,0)
m_2=(1,0,….,0)
m_3=(0,1,….,0)
m_4=(1,1,….,0)
m_5=(0,0,1,..,0)
…
m_{2^t}=(1,1,1,..,1)

2^t minterms

$\vec{m}_1$=(1,0,0,0,0,….,0)
$\vec{m}_2$=(0,1,0,0,0,….,0)
$\vec{m}_3$=(0,0,1,0,0,….,0)
$\vec{m}_4$=(0,0,0,1,0,….,0)
$\vec{m}_5$=(0,0,0,0,1,….,0)
…
$\vec{m}_{2^t}$=(0,0,0,0,0,….,1)

2^t minterm vectors

Points to the docs where only index terms k_1 and k_2 co-occur and the other index terms disappear

Point to the docs containing all the index terms

Pairwise orthogonal vectors $\vec{m}_i$ associated with minterms m_i as the **basis** for the ***generalized vector space***

Generalized Vector Model (cont.)

- Minterm vectors are pairwise orthogonal. But, this does not mean that the index terms are independent
  - Each minterm specifies a kind of dependence among index terms
  - That is, the co-occurrence of index terms inside docs in the collection induces dependencies among these index terms

Generalized Vector Model (cont.)

- The vector associated with the term k_i is represented by **summing** up all minterms containing it and **normalizing**

$$\vec{k}_i = \frac{\sum_{\forall r, g_i(m_r)=1} c_{i,r} \vec{m}_r}{\sqrt{\sum_{\forall r, g_i(m_r)=1} c_{i,r}^2}} = \sum_{\forall r, g_i(m_r)=1} \hat{c}_{i,r} \vec{m}_r$$

$$\text{where} \quad \hat{c}_{i,r} = \frac{c_{i,r}}{\sqrt{\sum_{\forall r, g_i(m_r)=1} c_{i,r}^2}}$$

$$c_{i,r} = \sum_{\boxed{d_j \mid g_l(\vec{d}_j) = g_l(m_r), \text{ for all } l}} w_{i,j}$$

All the docs whose term co-occurrence relation (pattern) can be represented as (exactly coincide with that of) minterm m_r

- The weight associated with the pair $[k_i, m_r]$ sums up the weights of the term k_i in all the docs which have a term occurrence pattern given by m_r.
- Notice that for a collection of size N, only N minterms affect the ranking (and not 2^N)

$g_i(m_r)$ Indicates the index term k_i is in the minterm m_r

Generalized Vector Model (cont.)

- The similarity between the query and doc is calculated in the space of minterm vectors

$$\vec{d}_j = \sum_i w_{i,j} \vec{k}_i \quad \Rightarrow \quad = \sum_r s_{j,r} \vec{m}_r$$

$$\vec{q}_j = \sum_i w_{i,q} \vec{k}_i \quad \Rightarrow \quad = \sum_r s_{q,r} \vec{m}_r$$

t-dimensional 2^t-dimensional

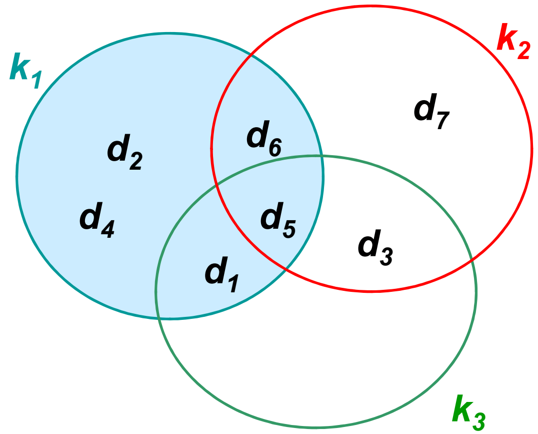
$$sim\left(\vec{q}_j, \vec{d}_j\right) = \frac{\sum_i w_{i,q} \cdot w_{i,j}}{\sqrt{\sum_i w_{i,q}} \sqrt{\sum_i w_{i,q}}}$$

$$sim\left(\vec{q}_j, \vec{d}_j\right) = \frac{\sum_r s_{q,r} \cdot s_{d,r}}{\sqrt{\sum_r s_{q,r}} \sqrt{\sum_r s_{d,r}}}$$

Generalized Vector Model (cont.)

- **Example** (a system with three index terms)

minterm	k_1	k_2	k_3
m_1	0	0	0
m_2	1	0	0
m_3	0	1	0
m_4	1	1	0
m_5	0	0	1
m_6	1	0	1
m_7	0	1	1
m_8	1	1	1



$$\vec{k}_1 = \frac{c_{1,2}\vec{m}_2 + c_{1,4}\vec{m}_4 + c_{1,6}\vec{m}_6 + c_{1,8}\vec{m}_8}{\sqrt{{c_{1,2}}^2 + {c_{1,4}}^2 + {c_{1,6}}^2 + {c_{1,8}}^2}}$$

$$\vec{k}_2 = \frac{c_{2,3}\vec{m}_3 + c_{2,4}\vec{m}_4 + c_{2,7}\vec{m}_7 + c_{2,8}\vec{m}_8}{\sqrt{{c_{2,3}}^2 + {c_{2,4}}^2 + {c_{2,7}}^2 + {c_{2,8}}^2}}$$

$$\vec{k}_3 = \frac{c_{3,5}\vec{m}_5 + c_{3,6}\vec{m}_6 + c_{3,7}\vec{m}_7 + c_{3,8}\vec{m}_8}{\sqrt{{c_{3,5}}^2 + {c_{3,6}}^2 + {c_{3,7}}^2 + {c_{3,8}}^2}}$$

	k_1	k_2	k_3	minterm
d_1	2	0	1	m_6
d_2	1	0	0	m_2
d_3	0	1	3	m_7
d_4	2	0	0	m_2
d_5	1	2	4	m_8
d_6	1	2	0	m_4
d_7	0	5	0	m_3
q	1	2	3	

$$c_{1,2} = w_{1.2} + w_{1.4} = 1 + 2 = 3$$
$$c_{1,4} = w_{1.6} = 1$$
$$c_{1,6} = w_{1,1} = 2$$
$$c_{1,8} = w_{1,5} = 1$$

$$\vec{k}_1 = \frac{3\vec{m}_2 + 1\vec{m}_4 + 2\vec{m}_6 + 1\vec{m}_8}{\sqrt{3^2 + 1^2 + 2^2 + 1^2}}$$

$$c_{2,3} = w_{2,7} = 5$$
$$c_{2,4} = w_{2,6} = 2$$
$$c_{2,7} = w_{2,3} = 1$$
$$c_{2,8} = w_{2,5} = 2$$

$$\vec{k}_2 = \frac{5\vec{m}_3 + 2\vec{m}_4 + 1\vec{m}_7 + 2\vec{m}_8}{\sqrt{5^2 + 2^2 + 1^2 + 2^2}}$$

$$c_{3,5} = 0$$
$$c_{3,6} = w_{3,1} = 1$$
$$c_{3,7} = w_{3,3} = 3$$
$$c_{3,8} = w_{3,5} = 4$$

$$\vec{k}_3 = \frac{0\vec{m}_5 + 1\vec{m}_6 + 3\vec{m}_7 + 4\vec{m}_8}{\sqrt{0^2 + 1^2 + 3^2 + 4^2}}$$

Generalized Vector Model (cont.)

- **Example**: Ranking

$$\vec{k}_1 = \frac{3\vec{m}_2 + 1\vec{m}_4 + 2\vec{m}_6 + 1\vec{m}_8}{\sqrt{3^2 + 1^2 + 2^2 + 1^2}} = \frac{3\vec{m}_2 + 1\vec{m}_4 + 2\vec{m}_6 + 1\vec{m}_8}{\sqrt{15}}$$

$$\vec{k}_2 = \frac{5\vec{m}_3 + 2\vec{m}_4 + 1\vec{m}_7 + 2\vec{m}_8}{\sqrt{5^2 + 2^2 + 1^2 + 2^2}} = \frac{5\vec{m}_3 + 2\vec{m}_4 + 1\vec{m}_7 + 2\vec{m}_8}{\sqrt{34}} \qquad \vec{k}_3 = \frac{0\vec{m}_5 + 1\vec{m}_6 + 3\vec{m}_7 + 4\vec{m}_8}{\sqrt{0^2 + 1^2 + 3^2 + 4^2}} = \frac{1\vec{m}_6 + 3\vec{m}_7 + 4\vec{m}_8}{\sqrt{26}}$$

$$\vec{d}_1 = 2\vec{k}_1 + 1\vec{k}_3$$

$$= \underbrace{\frac{2\cdot 3}{\sqrt{15}}}_{s_{d1,2}}\vec{m}_2 + \underbrace{\frac{2\cdot 1}{\sqrt{15}}}_{s_{d1,4}}\vec{m}_4 + \underbrace{\left(\frac{2\cdot 2}{\sqrt{15}} + \frac{1\cdot 1}{\sqrt{26}}\right)}_{s_{d1,6}}\vec{m}_6 + \underbrace{\frac{1\cdot 3}{\sqrt{26}}}_{s_{d1,7}}\vec{m}_7 + \underbrace{\left(\frac{2\cdot 1}{\sqrt{15}} + \frac{1\cdot 4}{\sqrt{26}}\right)}_{s_{d1,8}}\vec{m}_8$$

$$\vec{q} = 1\vec{k}_1 + 2\vec{k}_2 + 3\vec{k}_3$$

$$= \underbrace{\frac{1\cdot 3}{\sqrt{15}}}_{s_{q,2}}\vec{m}_2 + \underbrace{\frac{2\cdot 5}{\sqrt{34}}}_{s_{q,3}}\vec{m}_3 + \underbrace{\left(\frac{1\cdot 1}{\sqrt{15}} + \frac{2\cdot 2}{\sqrt{34}}\right)}_{s_{q,4}}\vec{m}_4 + \underbrace{\left(\frac{1\cdot 2}{\sqrt{15}} + \frac{3\cdot 1}{\sqrt{26}}\right)}_{s_{q,6}}\vec{m}_6 + \underbrace{\left(\frac{2\cdot 1}{\sqrt{34}} + \frac{3\cdot 3}{\sqrt{26}}\right)}_{s_{q,7}}\vec{m}_7 + \underbrace{\left(\frac{1\cdot 1}{\sqrt{15}} + \frac{2\cdot 2}{\sqrt{34}} + \frac{3\cdot 4}{\sqrt{26}}\right)}_{s_{q,8}}\vec{m}_8$$

$$sim(q,d) = \text{consine}(q,d) = \frac{\sum_{r | s_{q,r} \neq 0 \wedge s_{d,r} \neq 0} s_{q,r} \cdot s_{d,r}}{\sqrt{\sum_{r | s_{q,r} \neq 0 \wedge s_{d,r} \neq 0} s_{q,r}^2}\sqrt{\sum_{r | s_{q,r} \neq 0 \wedge s_{d,r} \neq 0} s_{d,r}^2}}$$

The similarity between the query and doc is calculated in the space of minterm vectors

$$sim(q,d_1) = \frac{s_{q,2}s_{d_1,2} + s_{q,4}s_{d_1,4} + s_{q,6}s_{d_1,6} + s_{q,7}s_{d_1,7} + s_{q,8}s_{d_1,8}}{\sqrt{s_{q,2}^2 + s_{q,3}^2 + s_{q,4}^2 + s_{q,6}^2 + s_{q,7}^2 + s_{q,8}^2}\sqrt{s_{d_1,2}^2 + s_{d_1,4}^2 + s_{d_1,6}^2 + s_{d_1,7}^2 + s_{d_1,8}^2}}$$

Generalized Vector Model (cont.)

- Term Correlation
  - The degree of correlation between the terms k_i and k_j can now be computed as

$$\vec{k}_i \bullet \vec{k}_j = \sum_{\forall r | g_i(m_r)=1 \wedge g_j(m_r)=1} \hat{c}_{i,r} \times \hat{c}_{j,r}$$

    - Do not need to be normalized? (because we have done it before! See p26)

More on Generalized Vector Model

- Advantages
  - Model considers correlations among index terms
  - Model does introduce interesting new ideas
- Disadvantages
  - Not clear in which situations it is superior to the standard vector model
  - Computation cost is fairly high with large collections
    - Since the number of “active” minterms might be proportional to the number of documents in the collection

Despite these drawbacks, the generalized vector model does introduce new ideas which are of importance from a theoretical point of view.